SECOND YEAR PERSONAL PURSUIT 2017/2018

THE ART AND SCIENCE OF OPTICAL ILLUSIONS

(INFORMALLY: A STUDY ON M.C. ESCHER)

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A VERY BRIEF INTRODUCTION TO ESCHER'S WORK

With more and more research, it becomes apparent that M.C. Escher was hardly seen as an artist, let alone a graphic artist, in his time. Looking at his pieces now, it is difficult for many of us not to call them works of art because of their beauty and the wonder that they ignite within us. Yet less than a century ago, art was still being analysed with a very blinkered mindset, where work that did not fit the descriptions of traditional art of centuries past was not deemed to be art. It was first the more scientifically minded who began to take an interest in Escher's work, appreciating the complexity and (mathematical) creativity of his drawings. Given the lack of



Drawing Hands, 1948

interest that there was in his work before the mathematicians arrived, it is even more remarkable that Escher continued in his own vein; he never yielded to the pressures of critics of his time, nor the financial difficulties that came along with not creating commercially popular works. And despite being called a promising print-maker of landscapes due to his technically impressive woodcuts, he instead persevered in making countless drawings and prints of unusual subject matter that would one day astound viewers across the world.



Alfedena Abruzzi, 1929

Escher began his career by making sketches and woodcuts of the southern European countrysides (predominantly Italy), but slowly turned towards visualising the ideas and thoughts that dominated his mind — questions

and puzzles that were constantly brewing within him. Although his landscape woodcuts showed great skill, they lacked the creativity that is very transparent in his now-famous works: that simply so distinctly "Escher" vibe of scenes with the common sense of a dream. However anyone who would dare call his pieces transcendental, or claim that there is any sort of depth as to the meaning, other than what can be directly perceived at first (or second) glance,

would not be appreciated by the artist himself. Escher wholeheartedly dismissed these sorts of art-critic comments, claiming that there was never any ulterior meaning behind his work. Funnily enough though, Escher repeatedly commented in multiple interviews that "drawing is deception", meaning he was aware of the psychological effects that his work had. His intention behind these words, were that drawings are inherently deceptive, in that they can portray three-dimensional worlds through use of a two-dimensional medium. That drawings are a suggestion of reality, and that the viewer wants to be tricked.



When looking through Escher's prints in chronological order, it is detectable that his subject focus went through a number of developments over the years. As already mentioned, the start of his career was primarily focused on landscapes, and thereafter he became intrigued by cyclic patterns so to say, where this



Cycle, 1938



Belvedere, 1958

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is no clear beginning or end to the picture. For example, *Drawing Hands* illustrates this well, as do many of his Metamorphosis prints, like *Cycle*. After this his attention shifted towards geometric shapes and patterns, including building structures and playing with viewer perception (see *Gravitation*, and *Belvedere*). In his final years, it seems as though a combination of his cyclic and



Ascending and Descending, 1960

geometric work brought forth new concepts, such as *Waterfall*, and *Ascending and Descending*, some of his most brilliant work.

With these varied interests and paths that he followed, each of his pieces keeps the viewer enjoying and wondering. So what is it in all of Escher's work that engages or tricks the mind so well? What are his techniques, how does he think? What sorts of mathematics, physics, psychology, and optics make up the work that we see?



Waterfall, 1961

THE MATHEMATICS OF ESCHER'S WORK

What intrigued me most when looking at Escher's work, is how he managed to make his curious ideas so mathematically 'correct'; for example, his drawings of mirrors and their reflections perplex the viewer, but are physically accurate representations of reality. Born in



Balcony, 1945

Leeuwarden in 1898, Escher did not study mathematics beyond high school, and did not have digital manipulation software as is common these days, in order to distort images properly; he had no way in which to create a bulge on top of a photo to then copy, but made the *Balcony* lithograph



Print Gallery, 1956

using his own calculations, although relying on geometry far more than pure mathematics. And what the mathematically correct centre of *Print Gallery* is, is a question very popular in non-Euclidean mathematics, and was solved by two Dutch mathematicians at the University of Leiden, Hendrik Lenstra and Bart de Smit.

PRINT GALLERY AND THE BALCONY

In Print Gallery, there is a man looking at a picture that depicts a line of buildings broaching a harbour, but within one of these buildings, is the gallery in which he is standing, and there is the man again, looking at the same picture. This is an example of the "Droste Effect", where a picture shows itself within the picture, thereby creating an infinite continuation of the same picture within a picture. "Droste" is a Dutch chocolate company that showed this effect on one of its boxes of



Droste cocoa packaging

chocolate powder, which is where the name for the effect originates.

The following is a simple explanation of how the mathematicians at the University of Leiden solved the problem of what lies at the centre of *Print Gallery*.¹ First, the landscape was reconstructed into its "original" form using the grid that Escher had used to distort it, but this time in reverse. This resulted in eight photos showing the cycle of the picture, due to the picture having been enlarged by a factor of 256 (2⁸). This means that



Escher's grid for mapping Print Gallery

each subsequent picture zooms in by a factor of 2 on the previous picture. These eight straightened pictures were then used to create a complete straightened version with all the blank spaces filled in, and lines connected seamlessly.



¹ B. de Smit and H. W. Lenstra Jr. (2003). The Mathematical Structure of Escher's Print Gallery. Artful Mathematics, Volume 5, Number 4, pp446-451 retrieved from http://www.ams.org/notices/200304/fea-escher.pdf



Grid construction for the Balcony, taken from Bruno Ernst's The Magic Mirror of M.C. Escher



Alternative version of Print Gallery as a double spiral.



Balcony, 1945

A much simpler piece to understand the construction of, is *Balcony*. Here Escher took a regular drawing he had made of a mediterranean view, and then split up the scene into a grid. He then drew a plain grid with a bulge in the centre through the use of curved lines, that had the same number of grid-squares as the scene's grid. After this, all he needed to do was draw each grid square in his original painting, within the bulged grid construction, ensuring that lines transcending individual grids remained continuous. After intensive manual labour, this results in the stunning fisheye effect that can be seen here.

NON-EUCLIDEAN GEOMETRY

After speaking with one of my supervisors, I delved into the theory of non-Euclidean geometry by reading some of the textbook *Euclidean and Non-Euclidean Geometries: Development and History* by Marvin Jay Greenberg,² specifically Chapter 1 introducing geometry, Chapter 6 on non-Euclidean geometry's history, and Chapter 8 about application and what this area of mathematics means for the world. Here is a short summary of my findings:

First of all, what is (non-)Euclidean geometry?

Forms of non-Euclidean geometry only began to be mathematically proven in the 1800s, with the name "non-Euclidean" coined by the German mathematician Carl Friedrich Gauss. In fact, many mathematicians were scared to report their findings/speculations, for fear of ridicule.

Euclidean geometry began with the great Greek mathematician Euclid (in around 300-200 BC), who was the first to develop a concept for geometry, the study of objects and their relative sizes and positions in space. Euclidean geometry is essentially the geometry that is taught to children in primary schools, and in much of high school too — it is a model of the world that works well over

² Greenberg, M. J. 2008. Euclidean and Non-Euclidean Geometries: Development and History. W. H. Freeman and Company, New York.

short distances, but the Theory of General Relativity depends on physical space with regard to time not being Euclidean i.e. its being non-Euclidean.

Euclidean geometry depends on defining distance and angles in a metric manner, so when these are removed, one can be left with two very well known types of non-Euclidean geometry: elliptic geometry, and hyperbolic geometry. A common and simple way to explain the difference between Euclidean, elliptic, and hyperbolic geometry, is as follows:

Consider two straight lines that are both perpendicular to a third straight line. In Euclidean geometry, these two lines remain parallel to one another indefinitely, i.e. the perpendicular distance between the two lines will always remain the same — see the figure Euclidean to the right for a representation of this.

Elliptic

In elliptic geometry, the two lines will begin to curve towards one another as their distance from the third line increases (and will cross one another at some point), and in hyperbolic geometry, the two lines will begin to curve away from one another, as their distance from the third line increases. There are numerous other types on non-Euclidean geometry in addition to these two. From these basic descriptions, one can likely extract

Hyperbolic

the perception that the easiest geometry to correctly draw something in, would be Euclidean geometry - nearly all traditional paintings follow this 'classic', 'realistic' style. Yet I am concerned with non-Euclidean geometry, because it is this type of geometry that Escher visualised so creatively well for mathematicians. In the real world, Euclidean geometry would seem to be the most useful of the different geometries, for example in construction, when taking measurements for a house. However, had it not been for non-Euclidean geometry, Albert Einstein would not have developed the Theory of General Relativity, and it also gave rise to Fuchsian and Kleinian groups, the Uniformisation of Compact Connected Orientable Surfaces, and more, as well as Escher's tessellations that visualised the hyperbolic plane.



Circle Limit I, 1958



Circle Limit IV, 1960

In order to understand these drawings in a physical sense, one might imagine standing within the prints themselves. Because both of these Circle Limit prints are drawn in the hyperbolic plane, when walking towards the edges of the drawings (i.e. to the outside of the circle), one can never reach the edge itself. Instead, one shrinks along with the picture, always staying the same size relative to the fish or the bats, which also means that there is no way to tell where one really is within the picture, because everything looks the same for an infinite distance.

TESSELLATIONS AND PERSPECTIVE

Tessellations are another type of mathematics that Escher made popular with the general public through his work — he even created tessellations in non-Euclidean geometries (see the Circle Limits above)! Tessellations occur when one divides up a plane into regular divisions without leaving any gaps. Shapes that can tesselate by themselves are triangles, squares, rectangles, and hexagons. With these basic shapes it is difficult to imagine that tessellations could ever be very interesting. One can also combine two or more shapes to create a tessellation. However, Escher brought tessellations to a



Day and Night, 1938

whole new level, devising tessellations in the shapes of animals, such as the *Day and Night*. And often made tessellations merge into different shapes, as can be seen in his Metamorphosis works.



Up and Down, 1947

Escher's love for tessellations is believed to have originated from his early visits to the Alhambra in Spain, where there are many geometric tessellated tile formations. Roger Penrose, the esteemed mathematician, is another famous creator of complex tessellations who inspired Escher, and it went the other way around too... Penrose and Escher began their correspondence after Penrose viewed an Escher exhibition in the Netherlands. Having seen Escher's impossible drawings, Penrose devised the tri-bar, popularly referred to as the "impossible triangle". After showing this to Escher, Escher in turn became inspired to create two of his most well-known, and newest, drawings, *Ascending and Descending*, and *Waterfall*, both of which show a looped scene. *Waterfall* is fantastic demonstration of using the Penrose triangle to create an optical illusion. When looking at the waterways, it slowly becomes apparent that they are the parts of two Penrose triangles hooked together.

Perspective is the key ingredient for these intricate drawings of what look like ordinary scenes in the first few milliseconds of glancing at them. A clear example is *Up and Down*, where Escher chooses unusual vanishing points (left and right at both the top and bottom of the drawing, and in the centre)

in order to make the same side of a wall or a set of stairs, be perceivable from different angles. In *Up and Down*, Escher could have drawn two different scenes at the top and bottom, to interconnect in the centre, but Escher (as ever) exaggerated the trick he was playing by using the same scene so that it looks like a game with mirrors.



Up and Down perspective sketch 7

PRINTMAKING AND AESTHETICS

What I personally find all the more impressive about Escher's artwork, is that many of his most complex pieces are not drawn by hand, but printed from woodcuts, an extremely time-consuming and delicate craft. Before the printmaking process began, Escher of course spent hours designing his works and thinking out his ideas on paper. When looking through the 'rough preliminary sketches' that would precede his final pieces, many of those sketches are remarkably detailed and interesting, and a fair percentage of creative ideas never made it to a print, as they did not interest Escher enough. After drawing out an extremely precise version of his concept, he would transfer this illustration to a block of wood (mirror image of course, probably using tracing paper), and then would carve the wood so that all the lines he wanted to have seen in his drawings, protruded at the right distance from the incised wood. Escher's color palette is probably the least remarkable part of his work: he always opted for either monochromatic, or dark, earthy tones of green, brick red, or brown. The reason for this is most likely that for every color Escher wanted to include in his print, he would need to use a separate block of wood, so that he could cover every block evenly with paint, without worrying about colors blending. Escher was also more concerned with showing his subject matter and the ideas behind his work, than creating a beautiful masterpiece, which could be a reason for why he stuck with woodcutting: he did not feel the need to impress people by creating visually appealing artwork, and therefore felt no need to opt for the easier medium of pencil or paint that could allow him to work faster.

It is Escher's talent for precision and realism that really helped a lot of his work to pop out to viewers. Where some of his drawings of animals were interesting distortions and fairly cartoon-like, much of his work involved very detailed drawing and woodcuts, to the extent that the viewer might be fooled in the first few seconds that it is a photograph. And this realism was in such stark contrast with the trickery he employed to make impossible scenes a (fake) reality.

For his subject matter, Escher often went back to the same muses in nature: birds, fish, and reptiles. It could be because the symmetry of these animals makes it easy to recognise them, even when having undergone a distortion to fit into a tessellation, for example. Symmetry also helps the viewer's eye be guided through the image, and this of great importance in Escher's works. Symmetry has long been linked to the realm of aesthetics, due the strong cognitive response it elicits in humans. Humans are very sensitive to (vertical) symmetry, because it portrays consistency and balance. It has a calming effect on the viewer, and people tend to be able to recognise symmetric figures in images, more quickly. There is no exact science to aesthetics, but some of the building blocks of aesthetics are: Lines, Space, Balance, Color, and Contrast, and here are some short explanations of where and how Escher demonstrated these in his works:

Lines: All of Escher's work was incredibly precise, his lines were neat, always drawn in the correct plane, and at exactly the correct angle, ensuring consistency and correct perspective.

Space: Escher hardly ever left any white space in his work. Although there is usually a lot of white space in traditionally aesthetic imagery, Escher's method of filling the entire canvas without a single gap, was an intelligent way of bypassing the need for any white space at all.

Balance: This refers to the symmetry already mentioned above.

Color: I have already touched upon the notable lack of color in Escher's works, but both his monochromatic and colored pieces usually have a sense of harmony within them.

Contrast: Escher's tessellations probably demonstrate this the best, where he capitalised on the dynamic effect of using two contrasting shades to represent different forms — see *Day and Night*.

MY OWN WORKS INSPIRED BY ESCHER



Becoming familiar with the Penrose triangle

Escher is an artist whose style has been imitated the world over, meaning that creating my own Escheresque paintings ran the risk of lacking originality, as thousands have broached different subject matters with the Escher style. I wanted to create three completely different styles of painting, all being a (different) kind of optical illusion. I wanted to have one painting of the Penrose Triangle or a similar illusion that involves that geometric impossibility. I wanted a second painting to be a more detailed optical illusion, playing with shapes, and hopefully one that might remind people of Escher. The third canvas was still up for debate, but I wanted it to be a completely different type of illusion.



Playing with interacting Penrose triangles

My process for the Penrose triangle began with a lot of concept sketches of the Penrose triangle, in order to become acquainted with how to draw the impossible shape. Then once I got the hang of the Penrose triangle, I was able to create many more similar shapes surprisingly easily (it just takes a change in the drawing mindset, and focusing on not letting oneself automatically draw what makes sense in the physical world. Later on, I looked at how I could interlock these different shapes to create even more complex imagery where the shapes conflicted with one another in dominance for the space.



I then tended towards hexagons and cubes, but still wanted to include the original Penrose triangle in my drawing, so I went for a combination of an interlocked half-finished cube, and a Penrose triangle. The triangle slips behind the cube at the top, but is in front at the bottom point. This would work if the cube were such that its centre extended out towards the viewer, but there are two geometric limitations here; firstly, the triangle's entire top edge lies behind the cube, rather than only the centre or the corners, and secondly, the cube is such that one cannot decide whether the centre is closer or farther away, and when it *is* closer, it means that the top corners of the triangle should be visible, which they are not. It is clearly quite complex to explain an optical illusion, so here a picture of the final piece probably *is* worth a thousand words.







Picture of entire canvas

The final painting was made using ink, a medium I wanted to experiment with, but was difficult to make smooth and keep within the lines. To clean up the image, I went over all the straight lines with a coat of gold, which both concealed the imperfections, and gave the image more interest and depth. I really like the impossible shapes that I created, as well as the composition and color, and no longer mind that the inks are not smooth, because it provides some texture.

My second piece took a lot more time, due to a very lengthy conception stage. Simply "coming up with an optical illusion" proved very difficult, and so I reached out to Escher's prints and again reviewed how he tessellated images and found shapes within space. I cannot point to an exact time when the idea was conceived, it was more of a progression that kept changing until the final stage — mainly because I found it so difficult to wrap my head around all the different lines I was drawing! The central concept of this piece, is about space, and how it can serve different agendas when looking upon from different angles/dimensions.



Initial sketches of final concept: Two arrows that occupy some of the same space



To the left I have tried to demonstrate the idea I had, with colors. The blue arrow is going away and up from the viewer, and when only considering the blue arrow, the space to the left of the arrow (in the nook between the arrow head and arrow body) should be empty. But this corner becomes the starting point for the downward-facing orange arrow with a slimmer body, and all of a sudden, the space that the viewer's eye initially considered to be empty, is now filled with mass. In real life, the blue and orange arrow could of course never co-exist like this.



I was finally happy with my concept, and then started preparing to transfer the design to canvas. I first of all created an abstract background with dark blue and green hues of acrylic paint, and then traced the design onto the dried surface. I then drew over all the lines with a golden pen (having liked the effect that it created with the first painting), and after that began filling in the squares with oil paints. I tried to go for a rose gold effect to complement the blue, but looking back, I regret having used oil paints because it is extremely difficult to work up a thick yet even layer of paint (I ought to have gone for acrylics).





I do like the concept of the clean, geometric shapes contrasting with the messy background, and especially like how the geometric shapes fade in and out of solidity as you travel diagonally across the canvas. After having drawn the entire row of arrowed spheres, I noticed that I could also tessellate them in other directions (except for leaving a very small gap between the circles), but decided against this because I did not want to run the risk of ruining the effect I had created

Second final piece: Arrows

When I asked people to take a look at this canvas, the responses always fell into two categories. The first would exclaim how mind-boggling the drawing was, and instantly see the idea I was trying to communicate: that of arrows occupying spaces that they physically could not. The second group of responses was that of silence, and a lot of head-tilting. It seems that there are quite a few people who require an explanation before they can see the illusion, and first only see a bundle of arrows. Some people said, "Oh, that's quite cool. You should 3D-print a model of this," without a hint of irony. They clearly could not see that this was a painting of something that could never be a physical object. Escher's paintings somehow always manage to clearly show the viewer the idea he had, but my canvas seems to be less clear to a lot of people. I think that this is down to the coloring of the arrows, which probably requires more contrast, as well as some shading for depth.





Like I have mentioned before, the third piece is one that I wanted to be completely different. I did not want it to have that geometric feel of the other two, but more a simple piece that might make people think about something in a different way. I tried a lot of tessellations similar to Escher's, including one I nearly went for with seals tessellated in one direction to the top-left-hand-corner, and fish swimming in the opposite direction to the bottom right. I then decided to go down a more creative direction than sticking with tessellations, and looked at how the same thing might look different from different mindsets. A sweet and not-so-subtle example of this is an ice

cream cone I drew with a melting scoop of vanilla ice cream on top. The melting dribbles of ice cream form some legs and a tail that lead up to a blissful cat taking a nap, rather than an ice cream scoop.

This then led me on to making more sketches of animals, including the watercolor sketch below of a whale spouting water from its blowhole. But out of the water comes another whale, and from that whale another. I found it a nice concept and so I developed it so that the smaller waves at first looked like they were just water (i.e. same color, and more abstract shape), and also wanted it so that the picture could be flipped upside down to show the same image. In order to do this I opted for a more cartoon-y feel so that the shapes could work in both orientations without looking strange. I was disappointed with the final result, because the cartoon effect was not how I had hoped it to be, and I rushed the development stage. If I were to do this again, I would try to instead make the whale more realistic, and have the background fade into a darker tone at both the top and bottom, in order to allow the smaller whales to fade into the background so that the whales on the bottom (in

both orientations) do not look strange. I would also make the first few "water whales" larger, so that I could add more detail as well as make the iteration clearer. This piece is by far my least favorite of the three, and my arrows paintings is definitely the highlight in my personal opinion.



Ice cream cat



Whale conception



Third final piece: Whale Conception

PERSONAL PURSUIT - END REFLECTION

As has probably become apparently after reading through this document, I have reflected a lot already throughout my work about what I was and was not satisfied with, my own thoughts on Escher's work, etc. But this final section here is a short reflection of the goals that I went into this Personal Pursuit with, where I fulfilled these goals, and where I fell short.

Learning Goals	Activities	Completion
Learn more about M.C. Escher as an artist, as well as the history of this interdisciplinary field of optical illusions. Know how the different disciplines of mathematics, psychology, and art interact to form a successful piece.	First I will read some books that give me a better starting ground and knowledge, specifically <i>Gödel</i> , <i>Escher</i> , <i>Bach: An Eternal Golden Braid</i> , by Douglas Hofstadter, and then <i>The</i> <i>Magic Mirror of M.C. Escher</i> , by Escher and Bruno Ernst.	I enjoyed reading both of these books, but <i>Gödel, Escher, Bach</i> did disappoint me in some senses (it did not live up to the high expectations), and was of little practical use to my project. The Magic Mirror of M.C. Escher was a really fun and interesting book that showed a lot of the practical techniques Escher used, and also gave a glimpse into his more personal thoughts.
	I will write a short history and summary on Escher, his life, and how he developed his art; the mathematics behind it, the psychological aspects, and the aesthetic value, as these are all intrinsic aspects that cannot explain the art on its own without help of the other aspects.	I did not delve a lot into the psychology of Escher's work, but did spend a substantial amount of time researching the mathematics, and read part of a textbook on non-Euclidean geometry. I did not want to write a detailed history on Escher himself, because I could not see the use in repeating what has been done before, but I liked my short introduction in this document, because it gave me clarity about how his style developed over time.
	Visiting the Escher Museum in den Haag.	I did this very early on in my Personal Pursuit, and found it to be a very enjoyable and informative trip!
	I will also read some books about Euclidean Geometry and then see how I can incorporate this knowledge into the designs I make.	I did not apply it very practically (and neither did Escher, it seems), but it was nice to get the theoretical basis for understanding how Escher's pieces worked. None of my own pieces used non-Euclidean geometries.

Develop my painting skills further through practice, applying them to a new context that will likely require great attention to perspective and shading.	After that I want to look at optical illusions (not necessarily just those of Escher) that are especially those focused on geometry (due to the mathematics involved), and paint some with my own twists – for example the Penrose Triangle.	I ended up sticking predominantly with Escher, and looking back, think that I could have branched out more, but I was just so fascinated with his style. Two of my three painting were very geometric- focused, and I did incorporate the Penrose triangle, which I adore. My painting skills themselves did not really improve in this Personal Pursuit because the shading was extremely basic, but I did get more practice with composition, and a lot of practice with using a ruler and protractor on a canvas!
Community	I will write pieces to go with the canvases, explaining the history or stories behind the illusions, and why it interests me.	I hope to be able to display my three canvases in the Citadel university building somewhere, along with complementary explanations (I think that the arrow piece works especially well for the many students who are still choosing which direction to take in their studies).

I loved the subject matter of this Personal Pursuit, and had a lot of fun researching Escher, and doing a lot of rough sketching throughout the year. The idea conception stage was what took by far the longest in this Personal Pursuit; a lot of time was spent with a sketchpad and pencil, drawing out random forms until an idea jumped out at me. The piece I take the most pride in, is my canvas of arrows, and I have surprised myself that I was able to make this — when I started out this Personal Pursuit, I was very hesitant to say that I would create my own optical illusions, because this is certainly far easier said than done. The piece I am most disappointed with, is obviously my whale painting, and wish I had the time to redo it, and not rush it (if there were one thing about this Personal Pursuit that I would change, it would be this painting. I still liked the concept, but the execution was poor). Generally I am pleased with this Personal Pursuit, and feel like the time I invested returned a creative outcome, and I hope that visitors to the Citadel will be able to enjoy my canvases!